CONCERNING THE EFFECT OF INTERPHASE HEAT EXCHANGE ON THE LONGITUDINAL THERMAL CONDUCTIVITY IN A GRANULAR LAYER

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Relations are derived for the temperature of elements in a granular layer and in a gas stream flowing against a uniform thermal flux, taking into account the interphase heat exchange.

In order to experimentally determine the longitudinal thermal conductivity in a granular layer ventilated by a gas stream, one usually applies the counterflow method to uniform thermal fluxes and gas streams [2, 3] and, for this purpose, a flat heater is installed where the gas exits from the layer.

The steady-state heat exchange for this process configuration is described by the equations:

$$\lambda_c \frac{\partial^2 t}{\partial x^2} + c_p G \frac{\partial t}{\partial x} = \alpha S (t - t_c); \tag{1}$$

$$\lambda_0 \frac{\partial^2 t_c}{\partial x^2} = \alpha S (t_c - t). \tag{2}$$

In dimensionless form we have

$$\frac{\partial^2 \Theta}{\partial Y^2} + C \frac{\partial \Theta}{\partial Y} = C (\Theta - \Theta_c); \tag{3}$$

$$\frac{\partial^2 \Theta_c}{\partial Y^2} = B (\Theta_c - \Theta), \tag{4}$$

where

$$Y = \frac{\alpha S}{c_p G} x = \frac{\mathrm{Nu_e}}{\mathrm{Re_e Pr}} \cdot \frac{4}{d_e} x; \tag{5}$$

$$B = \frac{(c_p G)^2}{\alpha S} \cdot \frac{1}{\lambda_0} = \frac{(\mathrm{Re_e Pr})^2}{\mathrm{Nue}} \cdot \frac{\varepsilon}{4\lambda_0}; \tag{6}$$

$$C = \frac{(c_p G)^2}{\alpha S} \cdot \frac{1}{\lambda c} = B \frac{\lambda_0}{\lambda c}; \tag{7}$$

$$\Theta = \frac{t - t_0}{t_{c,1} - t_0}; \quad \Theta_c = \frac{t_c - t_0}{t_{c.1} - t_0}.$$

The boundary conditions are

$$x = 0, \quad \Theta_c = 1; \quad x = \infty, \quad \Theta = \Theta_c = 0. \tag{8}$$

If the rate of interphase heat exchange is considerably higher than that of longitudinal heat transfer ($\alpha S \gg c_p G$), then Θ and ϑ_c were assumed close to one another so that Eq. (1) and (2) becomes

$$\lambda \frac{\partial^2 \Theta}{\partial x^2} = - c_p G \frac{\partial \Theta}{\partial x}, \tag{9}$$

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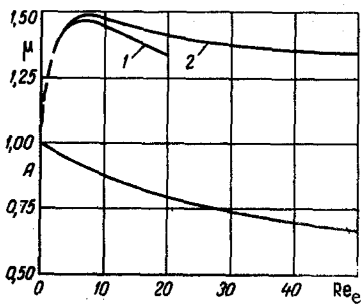

Fig. 1. Longitudinal convective heat conductivity in a granular layer, and ratio of gas elements and layer elements temperatures when gas and heat are counterflowing with interphase heat exchange taken into account: 1) $\bar{\lambda}_0 = 8$, 2) $\bar{\lambda}_0 = 13$.

where $\lambda = \lambda_0 + \lambda_c$. In this case

$$\Theta = \exp\left(-\frac{c_p G}{\lambda} x\right); \tag{10}$$

$$m \equiv \frac{\partial (\ln \Theta)}{\partial x} = -\frac{c_p G}{\lambda}. \tag{11}$$

The magnitude of λ was determined in [2, 3] according to Eq. (11). The solution to Eqs. (3) and (4) is then sought in the form:

$$\Theta_c = \exp(-KY); \tag{12}$$

$$\Theta = A\exp(-KY). \tag{13}$$

After inserting (12) and (13) into (3) and (4), we have

$$K^2 = B\left(1 - \frac{1}{1 + K - \frac{K^2}{C}}\right); \tag{14}$$

$$A = 1 - \frac{K^2}{B}. \tag{15}$$

From (12) and (13) we obtain

$$m = -KY_1, \tag{16}$$

where Y_1 is taken from (5) at x = 1 m. Taking into account (5) and (6), we rewrite expression (11) as

$$m = -BY_1 \frac{\lambda_0}{\lambda}. \tag{17}$$

Combining (16) and (17) will yield the ratio between the "apparent" value $\lambda_{c,\text{app}} = \lambda_c - \lambda_0$ found by test according to (10) and the real value λ_c:

$$\mu \equiv \frac{\lambda_{c,\text{app}}}{\lambda_c} = \left(\frac{B}{K} - 1\right)\frac{\lambda_0}{\lambda_c}. \tag{18}$$

In accordance with the data in [1] on interphase heat exchange and longitudinal convective diffusion, the following values were used for calculating μ:

$$\text{Re}_e = 1 - 100; \quad \text{Nu}_e = 0.63\text{Re}_e^{0.5}\text{Pr}^{0.33}; \tag{19}$$

$$\bar{\lambda}_c \approx 0.28 + 0.5\text{Re}_e\text{Pr}. \tag{20}$$

The values of B and C were determined from Eqs. (6) and (7) with Pr = 0.7 and $\varepsilon = 0.4$ for $\bar{\lambda}_0 = 8$ (glass) and 13 (steel), while K was found from Eq. (14). The results of calculations within the range of Re_e numbers encountered in tests [2] are shown in Fig. 1.

The conclusion drawn in [3] as to the difference between temperatures Θ and Θ_c being negligible is not accurate. The values of λ_c obtained in [2] are on the average 40% high. After appropriate corrections, they approach those calculated by Eq. (20). In Fig. 1 is also shown the ratio of temperatures $A = \Theta/\Theta_c$ calculated by Eq. (15); its magnitude is slightly dependent on $\bar{\lambda}_0$.

NOTATION

c_p	specific heat of gas;
d_e	equivalent diameter of granular layer;
G	mass rate of gas flow;
S	heat exchange surface per unit layer volume;
t	gas temperature;
t_c	temperature of layer elements;
t_0	temperature of gas at entrance to layer;
$t_{c,1}$	temperature of layer elements at $x = 0$;
x	linear coordinate in the direction of heat flow through the layer;
α	coefficient of heat exchange between layer and gas elements;
λ_c	longitudinal convective heat conductivity;
λ_0	thermal conductivity of unventilated layer;
$\bar{\lambda} = \lambda/\bar{\lambda}_g$;	
λ_g	thermal conductivity of gas.

LITERATURE CITED

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